

Gelbach in Logit: A Covariate Decomposition for the Logit Model applied to the Minimum Wage's Heterogeneous Impact

Miguel Salema
Supervisor: Pedro Raposo

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Gelbach in Logit

How can we divide the contribution of each control variable in explaining the effects of structural variables in a logit model?

The Problem

- No econometric meaning in adding variables sequentially;
- Gelbach (2016) solved it for OLS;
- Gelbach (2016) can't be directly used for logit.

Gelbach in Logit

Econometric Contribution: a Decomposition in the Logit Model

I adapt the Gelbach decomposition to the logit model using Lee (1980) OVB formula.
Omitted variable bias in the logit model has 2 components:

$$\text{logit } \hat{\text{bias}} = \underbrace{\text{confounding effects}}_{\substack{\text{Decomposable using Gelbach} \\ \text{between variables } z_1 \text{ and } z_2}} + \underbrace{\text{rescaling effects}}_{\text{Unexplained with Gelbach}}$$

Gelbach in Logit: Application

Heterogeneity of the Minimum Wage Effect on Separations

- 65% explained by firms;
- 28% explained by workers;
- 7% explained by tenure.

Ommited Variable Bias in the Logit Model

Example of a full model:

$$\ln \left(\frac{P^f(y_i = 1|x_i, z_{1i}, z_{2i})}{P^f(y_i = 0|x_i, z_{1i}, z_{2i})} \right) = \hat{\beta}_0^f + \hat{\beta}_1^f x_i + \hat{\beta}_2 z_{1i} + \hat{\beta}_3 z_{2i}$$

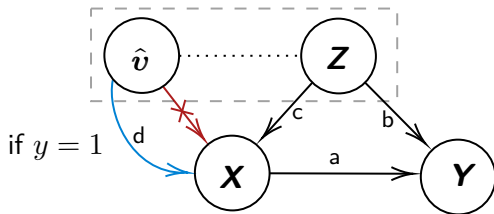
Example of a base model:

$$\ln \left(\frac{P^b(y_i = 1|x_i)}{P^b(y_i = 0|x_i)} \right) = \hat{\beta}_0^b + \hat{\beta}_1^b x_i$$

Omitted variable bias in the logit model:

$$\text{logit } \hat{bias}_x = \hat{\beta}_1^{base} - \hat{\beta}_1^{full} = \underbrace{\text{confounding effects}}_{\text{Decomposable using Gelbach between variables } z_1 \text{ and } z_2} + \underbrace{\text{rescaling effects}}_{\text{Unexplained with Gelbach}}$$

Diagram 1: Omitted Variable Bias in the Logit Model



1st auxiliary regressions:

$$z_{1i} = \hat{\Gamma}_0^{z_1} + \hat{\Gamma}_1^{z_1} x_i + \hat{v}_i^{z_1}$$

$$z_{2i} = \hat{\Gamma}_0^{z_2} + \hat{\Gamma}_1^{z_2} x_i + \hat{v}_i^{z_2}$$

The Logit Decomposition

Decomposing Confounding Effects: The Gelbach

1st auxiliary regressions:

$$z_{1i} = \hat{\Gamma}_0^{z_1} + \hat{\Gamma}_1^{z_1} x_i + \hat{v}_i^{z_1} ; \quad z_{2i} = \hat{\Gamma}_0^{z_2} + \hat{\Gamma}_1^{z_2} x_i + \hat{v}_i^{z_2}$$

$$\text{confounding bias}_{\beta_1} = \hat{\Gamma}_1^{z_1} \hat{\beta}_2 + \hat{\Gamma}_1^{z_2} \hat{\beta}_3$$

Decomposing Rescaling Effects

2nd auxiliary regressions:

$$\hat{v}_i^{z_1} = \hat{\delta}_0^{z_1} + \hat{\delta}_1^{z_1} x_i + \hat{\delta}_2^{z_1} y_i + \hat{e}_i^{z_1} ; \quad \hat{v}_i^{z_2} = \hat{\delta}_0^{z_2} + \hat{\delta}_1^{z_2} x_i + \hat{\delta}_2^{z_2} y_i + \hat{e}_i^{z_2}$$

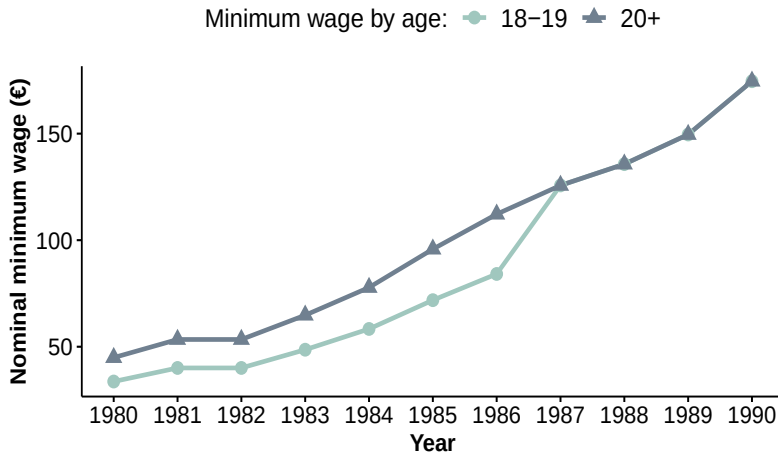
Assuming the residuals $\hat{v}$ are independent and conditionally normally distributed:

$$\text{rescaling bias}_{\beta_1} = \hat{\delta}_1^{z_1} \hat{\beta}_2 + \hat{\delta}_1^{z_2} \hat{\beta}_3$$

An Application

What are the main determinants of the minimum wage effect on separations?

Figure 1: A Minimum Wage Natural Experiment in Portugal



Minimum Wage Literature

Using this Natural Experiment:

- Pereira (2003);
- Portugal and Cardoso (2006);
- Cerejeira (2008).

Studies of the Minimum Wage Heterogeneous Effects:

- Harasztsi and Lindner (2019): by age, sector, and firm size;
- Cengiz et al. (2019): through the wage distribution;
- Burkhauser et al. (1996): by household income.

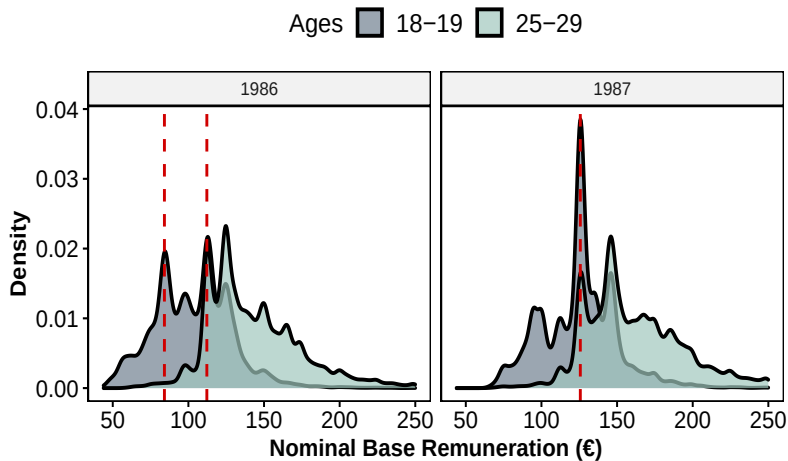
However, no study divides the heterogeneity.

Data

Quadros de Pessoal

- Census to all private firms with at least one employee in Portugal;
- Linked-employer-employee-dataset;
- Mandatory response by law.

Figure 2: The Effect of the Minimum Wage on the Wage Distribution



Triple Difference Method

$$\ln \left(\frac{Pr(y_{it} = 1|\mathbf{X})}{1 - Pr(y_{it} = 1|\mathbf{X})} \right) = \sum_{t=1987}^{2019} [\beta_{1t}t + \beta_{2t}teen_{it} + \beta_{3t}mw_{it} + \beta_{4t}(t \times teen_{it}) + \beta_{5t}(t \times mw_{it}) + \beta_{6t}(teen_{it} \times mw_{it}) + \alpha_t(t \times teen_{it} \times mw_{it})] + \xi_1 \ln(firm\ size_{J(it)t}) + \xi_2 tenure_{it} + \theta_i + \psi_{J(it)} + \varepsilon_{it}$$

t : Discrete variable of years from 1986 to 2019

i : Worker ID

J : Firm ID

$teen_{it}$: Dummy variable, 1 if 18-19 years old, 0 if 25-29

mw_{it} : Dummy variable, 1 if a worker is on the MW, 0 if on the 40 percentile above MW

$firm\ size_{J(it)t}$: Number of employees in the firm

$tenure_{it}$: Number of months working in the J firm

θ_i : Worker fixed effects

$\psi_{J(it)}$: Firm fixed effects

$\mathbf{X}$: Matrix of all explanatory variables

$y_{it} \begin{cases} 1 & \text{, if the worker separated in } t + 1 \\ 0 & \text{, if the worker didn't separate in } t + 1 \end{cases}$

Table 1: Descriptive Statistics

Bins			Separations		ln(firm size)		Tenure		Deleted Obs. for the logit TWFE (%)				Worker Obs.	
Year	Wage	Age	mean	sd	mean	sd	mean	sd	Total	Singleton	Perfectly Classified	Separation Singleton	mean	sd
1986	MW	18-19	0.27	0.45	3.66	1.48	27.20	18.44	15.21	9.87	5.93	24.23	15.24	9.37
1986	MW	25-29	0.25	0.43	4.41	1.74	80.28	60.51	12.71	8.27	4.84	22.88	14.08	8.82
1986	Above	18-19	0.25	0.43	4.45	1.71	35.80	41.28	18.78	11.26	8.47	27.89	14.07	9.62
1986	Above	25-29	0.22	0.41	5.60	2.35	73.91	54.34	9.90	6.85	3.26	23.34	15.97	9.79
1988	MW	18-19	0.36	0.48	4.24	1.54	26.49	20.25	15.51	10.29	5.82	23.05	14.48	9.05
1988	MW	25-29	0.39	0.49	3.84	1.66	53.45	52.44	11.12	6.64	4.80	14.79	14.68	8.94
1988	Above	18-19	0.38	0.49	4.47	1.73	30.48	42.18	16.57	10.65	6.63	23.02	14.23	9.39
1988	Above	25-29	0.32	0.47	5.09	2.23	62.47	50.82	6.92	4.46	2.58	12.91	16.87	9.72

Table 2: Average Minimum Wage Effect on Separations

Dependent Variable: Separations	Logit Models		APE from
	Base model	Full model	Full Model
$mw \times teen \times 1988$	0.36* [0.24; 0.47]	0.48* [0.34; 0.63]	0.06* [0.03; 0.08]
$\log(firm\ size)$	-	-0.02* [-0.02; -0.02]	-0.004* [-0.004; -0.004]
$tenure$	-	-0.00* [-0.00; -0.00]	-0.0004* [-0.0004; -0.0004]
(...)			
Worker FE	No	Yes	Yes
Firm FE	No	Yes	Yes
Deviance	1152272.74	61593462.09	-
Num. obs.	59260884	59260884	-
Num. of workers	-	579236	-
Num. of firms	-	678054	-

Table 3: Minimum Wage Displacement Effects Decomposed

	Model's Coeffs		True Bias		Decomposition										
					Confounding Effects: Gelbach					Rescaling Effects					Total
	Base (1)	Full (2)	Total (3)	Rescal. (4)	Worker (5)	Firm (6)	Size (7)	Tenure (8)	Sum (9)	Worker (10)	Firm (11)	Size (12)	Tenure (13)	Sum (14)	
α_{88}	0.36	0.48	−0.12	−0.10	−0.10	0.02	0.01	0.05	−0.02	−0.03	−0.06	0.00	−0.01	−0.09	−0.11
% bias			(100%)	(83%)	(81%)	(−17%)	(−7%)	(−41%)	(17%)	(22%)	(51%)	(0%)	(6%)	(79%)	(95%)
% Rescal.										(28%)	(65%)	(0%)	(7%)	(100%)	

Conclusion

In this Thesis:

- Develop a novel conditional covariate decomposition for the logit model;
- Divide the heterogeneity in the treatment effect;
- Find the firm to be the primary determinant of the minimum wage displacement effects.

Useful for Policy Evaluation

The logit decomposition is able to detect which factors exacerbate the nefarious/beneficial effects of a policy.

Thank you

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Miguel Salema

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